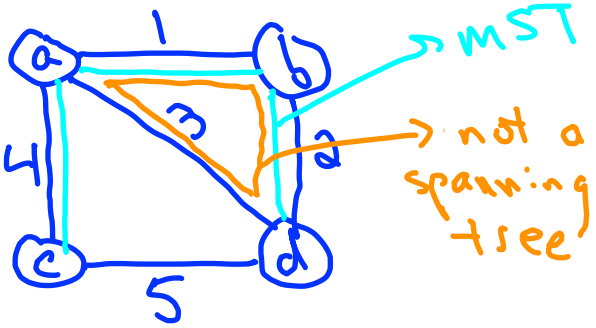
Section 19.1:
MST vs. TSP:
An Algorithmic Mystery

The Minimum Spanning Tree Problem

Input: Connected undirected graph $G=(V,E)$, real-valued cost c_e for each $e \in E$.

Output: A spanning tree $T \subseteq E$ with the minimum-possible total cost $\sum_{e \in T} c_e$.

[can be up to n^{n-2} spanning trees] (n = # of vertices)

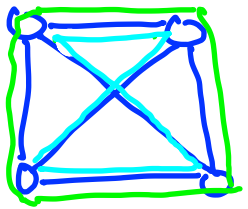


Prim / Kruskal: $O((m+n)\log n)$ time. (m = # of edges)

The Traveling Salesman Problem

Input: Complete undirected graph $G = (V, E)$, real-valued cost c_e for each $e \in E$.

Output: A tour $T \subseteq E$ of G with the minimum-possible total cost $\sum_{e \in T} c_e$.



Quiz #1

In an instance of TSP with $n \geq 3$ vertices, how many distinct tours are there?

(a) 2^n

(b) $\frac{1}{2}(n-1)!$

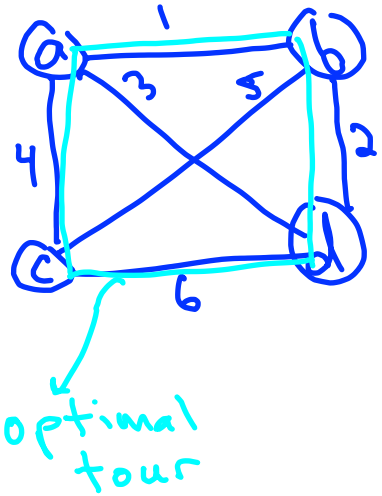
(c) $(n-1)!$

(d) $n!$

Quiz #2

What is the minimum sum of edge costs of a tour in the graph below?

(a) 12
(b) 13
(c) 14
(d) 15



Trying and Failing to Solve the TSP

- applications: ordering problems (tasks, genome fragments, etc.)
  - reference: Applegate et al. (The TSP: A Computational Study)

Fact: As of 2020, no known fast algorithm for the TSP!

Speculation: (Edmonds, 1967) No fast algorithm for the TSP exists.