Section 24.2: Greedy Heuristics for Buying Back Licenses

Four Temporary Simplifying Assumptions

- license = confers right to broadcast in a specified geographic area with little to no interference
  - specific channel assignment not part of property rights

("Middle Class Tax Relief + Job Creation Act")

Simplifying assumptions (to be removed)

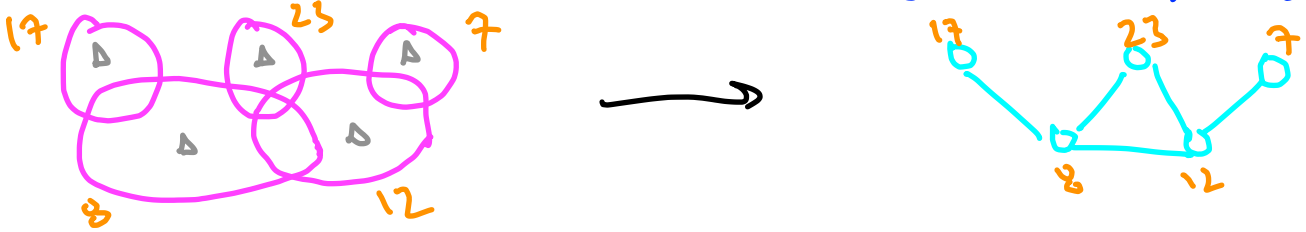
1. All stations that remain on air broadcast on a common channel.
2. Stations A, B can broadcast on same channel $\Longleftrightarrow$ no overlap in broadcasting regions
3. Known value for each station.
4. Government can unilaterally decide who stays on the air.

Ambushed by Weighted Independent Set

Goal: maximize total value of the stations that remain on the air, subject to no interference (i.e., disjoint broadcasting regions).

Diagnosis: exactly the weighted independent set problem! (NP-hard)

[vertices = stations, edges = overlapping stations, weights = station values]



Problem size: 1000s of vertices, tens of 1000s of edges

=> too big to be solved exactly, even with latest + greatest MIP solvers

Greedy Heuristic Algorithms

Weighted independent set: given undirected graph $G = (V, E)$ + weight $w_v \geq 0$ $\forall v \in V$, compute max-weight indep. set $S \subseteq V$ (mutually non-adjacent vertices)

Basic Greedy

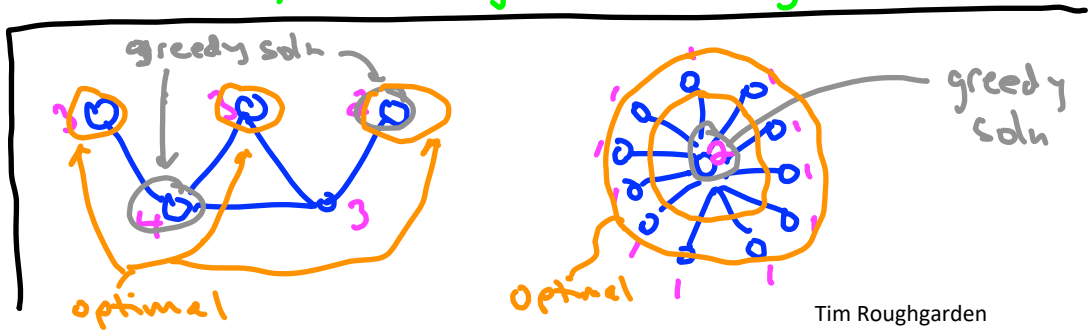
$S := \emptyset$

for each vertex $v \in V$, in nonincreasing order of weight w_v:

  if $S \cup \{v\}$ is feasible: (ie, v not adjacent to any $w \in V$)

    $S := S \cup \{v\}$

return S



Vertex-Specific Multipliers

Modification: order vertices in nonincreasing order of $\frac{w_v}{(1+\deg(v))}$ ("bang per buck")

(to discriminate against high-degree vertices)

$1+\deg(v)$ = # of vertices removed from future consideration

General Greedy

Compute β_v for each $v \in V$ [ex: $\beta_v = 1+\deg(v)$]

$S := \emptyset$

for each vertex $v \in V$, in nonincreasing order of w_v/β_v:

  if $S \cup \{v\}$ is feasible [v not adjacent to any $u \in S$]

    $S := S \cup \{v\}$

return S

In FCC Incentive Auction:

$$\beta_v := \sqrt{\deg(v)} \cdot \sqrt{pop(v)}$$

The Multi-Channel Case

Assume: still-on-air stations can be assigned one of k channels.

FCC Greedy

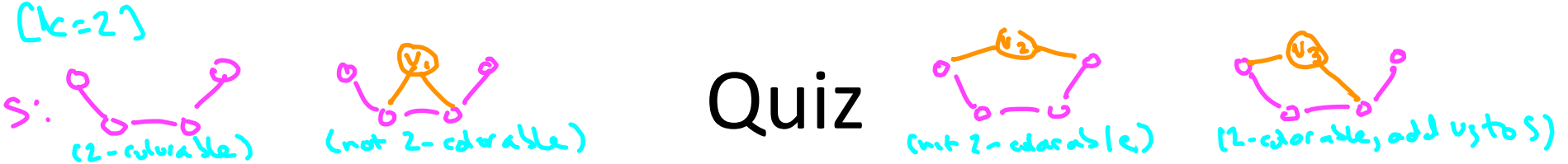
- compute β_v for each station [ex: $\beta_v = \sqrt{deg(v)} \cdot \sqrt{pop(v)}$]
- $S = \emptyset$
- for each station, in nondecreasing order of w_v/β_v:
  - if $S \cup \{v\}$ is feasible: [fit on k channels]
    - $S := S \cup \{v\}$
- return S

diagnosis: graph coloring!!

Ex:

[feasible if $k=3$ but not if $k=2$]

[3-colorable but not 2-colorable]



Quiz

Consider the sequence of feasibility-checking instances in the FCC Greedy algorithm. Which of the following are true?

[Choose all that apply.]

(a) If the instance in one iteration is feasible, so is the one in the next iteration.

(b) If the instance in one iteration is infeasible, so is the one in the next iteration.

(c) Each instance has one more station than the previous one.

(d) Each instance has one more station than the previous feasible one.