Section 21.5: Satisfiability Solvers

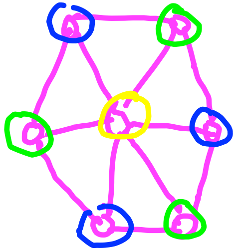
Example: Graph Coloring

Input: undirected graph $G=(V,E)$, positive integer k.

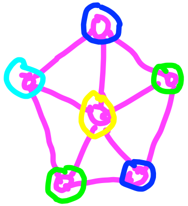
Output: A k-coloring of G, or a correct declaration that none exist.

Famous result: "Four Color Theorem"

3-colorable



not 3-colorable



Satisfiability

Idea: specify constraints with logic rather arithmetic.

Decision variables: Boolean (true/false).

Truth assignment: assignment of T/F to each variable. (2^n total, where n = # vars)

Constraints: disjunctions of literals.

logical OR NOT

Example: $x_1 \vee \neg x_2 \vee x_3$

(satisfied by 7 of 8 possible truth assignments to x_1, x_2, x_3)

Input: Boolean decision variables $x_1, x_2, \ldots, x_n$; list of m constraints, each a disjunction of one or more literals.

Output: truth assignment that satisfies all constraints (or correctly declare that none exist).

Encoding Graph Coloring as SAT

Variables: x_{vi} for each vertex $v \in V$ and color $i \in \{1, 2, \ldots, k\}$

(intended semantics: x_{vi} = true $\Longleftrightarrow$ v assigned color i)

Constraints: (1) $\underbrace{\neg x_{vi} \vee \neg x_{wi}}$ for all $(v,w) \in E$ and $i \in \{1, 2, \ldots, k\}$

↳ rules out assigning color i to both v & w

(2) $x_{v1} \vee x_{v2} \vee \ldots \vee x_{vk}$ for all $v \in V$

Encoding Graph Coloring as SAT

Latest & greatest solvers

Check www.satcompetition.org

Next level: satisfiability modulo theories (SMT) solvers (e.g., MSoft's z3)

```
p cnf 6 9
1 4 0
2 5 0
3 6 0
-1 -2 0
-4 -5 0
-1 -3 0
-4 -6 0
-2 -3 0
-5 -6 0
```

in file format expected by e.g. MiniSAT