Section 22.2:
3-SAT and the
Cook-Levin Theorem

The Cook-Levin Theorem

Cook-Levin Theorem: 3-SAT is NP-hard. (≈1971)

[Karp - showed full power of two-step recipe - 21 NP-hard problems] (1972)

[2-SAT solvable in linear time]

[no contradiction with semi-reliable SAT solvers]

The 3-SAT Problem

Input: Boolean variables $x_1, x_2, \ldots, x_n$; m constraints, each a disjunction of at most three literals.

Output: A truth assignment to $x_1, x_2, \ldots, x_n$ that satisfies every constraint (or correctly report that none exist).

logical OR

logical NOT

Ex:

$$\begin{cases} x_1 \vee x_2 \vee x_3 \quad & x_1 \vee \neg x_2 \vee x_3 \quad & \neg x_1 \vee \neg x_2 \vee x_3 \quad & x_1 \vee \neg x_2 \vee \neg x_3 \\ \neg x_1 \vee x_2 \vee x_3 \quad & x_1 \vee x_2 \vee \neg x_3 \quad & \neg x_1 \vee x_2 \vee \neg x_3 \quad & \neg x_1 \vee \neg x_2 \vee \neg x_3 \end{cases}$$

unsatisfiable [but satisfiable if any one constraint omitted]